P425/1

PURE MATHEMATICS

Paper 1

June/July. 2022

3 hours

TRIGONOMETRY TEST 2022

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section A and any **five** from section **B**.*

*Any additional question(s) answered will **not** be marked.*

***All** necessary working **must** be shown clearly.*

Begin each answer on a fresh page.

Silent non- programmable scientific calculators and mathematical tables with a list of formulae may be used.

TURN OVER

SECTION A: (40 MARKS)

*Attempt **all** questions in this section.*

1. Given that $cotA = \frac{4}{3}$ and $secB = \frac{17}{15}$ where A and B are both reflex angles. Find without using mathematical tables or calculator the value of $\tan(A - B)$. (05 marks)

2. Solve the equation: $8sin^2(\theta - 30^0) = 1 + cos2\theta$ for $0^0 \leq \theta \leq 360^0$.

   (05 marks)

3. Without using mathematical tables or calculator, prove that $cos165^0 + sin165^0 = cos135^0$. *(05 marks)*

4. Prove that $\frac{tanA+secA-1}{tanA-secA+1} = secA + tanA$. *(05 marks)*

5. Solve the equation: $3sinx + cos2x = 2$ for $-180^0 \leq x \leq 180^0$.

   (05 marks)

6. Solve the equation: $2cos^2\left(x - \frac{\pi}{2}\right) - 3cos\left(x - \frac{\pi}{2}\right) + 1 = 0$ for $0 \leq x \leq 2\pi$. *(05 marks)*

7. Solve: $sinx + sin2x + sin3x = 0$ for $0^0 \leq x \leq 180^0$. *(05 marks)*

8. Show that $\frac{sin3A}{1+2cos2A} = sinA$. Hence show that $sin15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

   (05 marks)

SECTION B: (60 MARKS)

*Attempt only **five** questions in this section.*

9. (a) Show that if $tan\frac{\theta}{2} = t$, $sin\theta = \frac{2t}{1+t^2}$ and $cos\theta = \frac{1-t^2}{1+t^2}$. Hence solve the equation $3cosx - 5sinx = 2$ for $0^0 \leq x \leq 360^0$. *(07 marks)*

(b) Solve the equation: $5cos\theta sin2\theta + 4sin^2\theta = 4$ for $0^0 \leq \theta \leq 360^0$.

(05 marks)

10.(a) Given $x = tan\theta - sin\theta, y = tan\theta + sin\theta$, show that $(x^2 - y^2)^2 - 16xy = 0$. *(05 marks)*

(b) Solve: $4sinxcos2xsin3x = 1$ for $0^0 \leq x \leq 180^0$. *(07 marks)*

11.(a) Show that $\frac{sin8\theta cos\theta - sin6\theta cos3\theta}{cos2\theta cos\theta - sin3\theta sin4\theta} = tan2\theta$. *(05 marks)*

(b)Given $ksinx = sin(x - \alpha)$, find $tanx$ in terms of k and α. Hence solve the equation $2sinx = sin(x - 60^0)$ for $0^0 \leq x \leq 360^0$.

(07 marks)

12.Express $12cos^2x - 16sinxcosx - 7$ in the form $a + bcos(2x + \alpha)$ where a and b are constants and α is a positive acute angle. Hence;

(a) Solve the equation: $12cos^2x - 16sinxcosx - 5 = 0$ for $0^0 \leq x \leq 360^0$. *(08 marks)*

(b) Find the maximum value of $\frac{1}{12cos^2x - 16sinxcosx - 5}$ and state the smallest positive value of x when it occurs. *(04 marks)*

13.(a) Show that if P,Q and R are angles of a triangle then $1 + cos2R - cos2P - cos2Q = 4sinPsinQcosR$. *(05 marks)* (b)Solve for x: $sin(x + 30^0) + sin(x + 60^0) = cos(x + 45^0) + cos(x + 75^0)$ for $0^0 \leq x \leq 360^0$. *(07 marks)*

14.(a) Prove that $tan(A - B) = \frac{tanA - tanB}{1 + tanAtanB}$. Hence solve the equation $tan(\theta - 45^0) = 6tan\theta$ for $-180^0 \leq \theta \leq 180^0$. *(07 marks)*

(b)The acute angle α is such that $tan\left(\alpha + \frac{\pi}{4}\right) = 41$, show that $cos\alpha = \frac{21}{29}$. Hence find $sin\alpha$. *(05 marks)*

15.(a) Prove that $sin4\theta = \frac{4tan\theta - 4tan^3\theta}{(1 + tan^2\theta)^2}$. Hence solve for t if $t = tan\theta$ given $t^4 + 8t^3 + 2t^2 - 8t + 1 = 0$ correct to 3 significant figures.

(08 marks)

(b) Show that $\sqrt{\frac{1 - sin\theta}{1 + sin\theta}} = sec\theta - tan\theta$. *(04 marks).*

16.Prove that

(a) $sin^4\theta + cos^4\theta = \frac{1}{4}(3 + cos4\theta)$. *(03 marks)*

(b) $cos^6x + sin^6x = 1 - \frac{3}{4}sin^2 2x$. *(04 marks)*

(c) $sin(A - B) + cos(A - B)tanC = sin2BsecC$ if A, B and C are angles of a triangle. *(05 marks)*

GOOD LUCK